Comparison of PIC and SIC with Lattice Reduction cancellation schemes for V-BLAST MIMO system

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Abstract—This paper discusses the design of the MIMO signal demodulation algorithm built according to the V-BLAST scheme. Several algorithms such as the Maximum Likelihood algorithm and QR-M algorithm, Minimum of Mean Square Error (MMSE), Successive Interference Cancellation (SIC) with MMSE (SIC-MMSE), Parallel Interference Cancellation (PIC) with MMSE (PIC-MMSE), Lattice Reduction MMSE (LR-MMSE), as well as modifications of SIC-LR-MMSE, PIC-LR-MMSE are considered. It is shown that the SIC-LR-MMSE and PIC-LR-MMSE algorithms, presented in this paper, having a linear computational complexity depending on the number of antennas and modulation order, provides good performance close to potentially achievable (MLA), which makes its practical use possible. The presented algorithm can be used to improve performance of CDN (Content Delivery Networks) or CMS (Content Management Systems) to transmit content in different distributed content delivery systems.

Keywords— MIMO, V-BLAST, MLA, QRM, MMSE, Lattice Reduction, SIC, PIC, CMS, CDN.

I. INTRODUCTION

This document is related to the demodulation of the user MIMO (Multiple Input – Multiple Output) signal in mobile networks using Vertical Bell Laboratories Layered Space-Time (V-BLAST) [1] and Orthogonal Frequency Division Multiplexing (OFDM) subscriber multiplexing [2]. A great number of various demodulation and accompanying algorithms relating to such signal are known from the literature [3-7].

One of the major algorithms is the Maximum Likelihood Method (MLA). This algorithm determines the potentially achievable characteristics. However, it cannot be implemented in practice due to its high computational complexity, which grows exponentially depending on the number of the degrees of freedom (number of antennas and modulation order).

Therefore, an abundance of other, simpler algorithms are proposed in the literature. The most well-known is the Minimum of Mean Square Error (MMSE) algorithm [2], which has linear complexity but, according to the analysis provided in this paper, is sufficiently inferior to the MLA algorithm in terms of the error rate $2 \cdot 10^{-2}$ (see simulation results).

To solve the above problem, different versions of the MLA algorithm using QR-decomposition of the channel matrix (QRM algorithm) have been investigated. This algorithm, as well as MLA, is exhaustive, but the maximum number of the degrees of freedom at each search iteration is fixed and does not exceed some pre-selected value M (thus the name of the algorithm). This algorithm demonstrates the performance close to potentially achievable, however it still has high implementation complexity by its very nature.

An important milestone in solving this problem is a Successive Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC) algorithms [4,5]. However its direct use with MMSE (see Fig. 2) provides only some gain, i.e. 2 dB under these simulation conditions. The remaining considerably big gain is still unattainable.

On the other hand, a Lattice Reduction [8] (LR-MMSE) algorithm using the channel matrix expansion in terms of the orthogonal basis is also known from the literature. This allows enhancing the MMSE performance by about 4 dB, which is inferior to the MLA algorithm by a significant value (about 1.5-2 dB as part of the present simulation).

Combining three algorithms SIC-LR-MMSE allows coming close to the potentially achievable performance as much as possible [9]. But during some hardware implementations SIC scheme might be less convenient than PIC scheme. This question is the main purpose of current research.

This paper studies the latter algorithm in more detail, provides its mathematical description and simulation results.

II. STATEMENT OF THE PROBLEM

Let us consider a fading channel in the OFDM system with one user built according to the V-BLAST scheme (Fig. 1):

$$z = Hs + n \quad (1)$$

Here $z$ is a received signal, $N$ is the number of transmitting antennas and $L$ is the number of receiving ones. $H$ is a channel matrix; let’s consider it as known during the current research.

$$z, n \in \text{Vect}(L, C), s \in \text{Vect}(N, C)$$

where $C$ is the set of complex numbers.

![Fig. 1. MIMO V-BLAST scheme.](image-url)
Modulation means that components of $s$ belong to the finite discrete constellation. 

$s \in C^N$

For example, QPSK:

$$\left\{ \frac{\pm 1}{\sqrt{2}}; \pm \frac{i}{\sqrt{2}} \right\}$$

16QAM:

$$\left\{ \pm \frac{1}{\sqrt{10}}; \pm \frac{i}{\sqrt{10}} \right\} \cup \left\{ \pm \frac{3}{\sqrt{10}}; \pm \frac{3i}{\sqrt{10}} \right\}$$

64QAM:

$$\left\{ \pm \frac{1,3,5,7}{\sqrt{42}}; \pm \frac{1,3,5,7}{\sqrt{42}} \right\}$$

Indeed, for M-QAM modulation, $M = 4m^2$ (and on condition that $M = 2^k$).

III. SIGNAL TRANSFORMATION AND ALGORITHMS DESCRIPTION

Let us introduce $A = \sqrt{\frac{3}{2(M-1)}}$

In that case $C^N = \{ s = a + ib; a, b \in \{ \pm A, \pm 3A, \ldots, \pm (2m-1)A \}\}$

According to this reason any part of constellation to the set of integers $\bar{s} = \{ (0,1, \ldots, 2m-1), (0,1, \ldots, 2m-1) \}$ can be done by

$$\bar{s} = \frac{1}{2A} \bar{s} + \frac{(2m-1)}{2} (1 + i) e_0$$

(3)

where $e_0 = (1, ..., 1)^T$

with backward transition

$$s = 2A \left( \bar{s} - 2m-1 \right) (1 + i) e_0$$

(4)

with corresponding transformation of $\bar{z}$:

$$\bar{z} = \frac{1}{2} \bar{z} + \frac{(2m-1)}{2} (1 + i) H e_0$$

(5)

In that case equation (1) can be rewritten as

$$\bar{z} = H \bar{s} + \bar{n}$$

(6)

with $(\bar{n}, \bar{n}) = \left( \frac{1}{2A} \right)^2 (n, n)$

Let us consider a few basic algorithms.

MLA

$s = \text{arg min}_{s \in C^N} (||z - Hs||)$

determines the potentially allowed interference immunity.

MMSE

$$s = W_{MMSE} \bar{z}$$

(7)

where

$$W_{MMSE} = H^+(HH^+ + (n,n))^{-1}$$

QRM

determines the potentially allowed interference immunity under the conditions when MLA cannot be used due to excessively high computational complexity. It was built according to the guidelines provided in [10].

Base idea of algorithm is to switch detection from full form of (1) to QR decomposition of channel matrix. Then, for each iteration, only some part of best replicas, are saved and used for later iterations.

LR-MMSE
Motivation of lattice reduction MMSE is to find matrix $T$ which makes $H$ orthogonal or nearly orthogonal

$$H = H^{\text{red}} T^{-1}$$

$$z = Hs + n = H^{\text{red}} T^{-1} s + n = H^{\text{red}} c + n$$

$$c = T^{-1} s$$

Algorithm of calculating $H^{\text{red}} T^{-1}$ is known from many articles as the Lenstra-Lenstra-Lovasz algorithm. We will not discuss this algorithm here, it is described in many papers, for example, [8].

Therefore to calculate $s$ estimation in the LR-MMSE algorithm, transformation should be performed (5), then calculate as

$$c = [W_{LR-MMSE} \bar{z}]^T$$

(8)

where $\lfloor \cdot \rfloor$ is a round down operation.

Then $\bar{s} = Tc$ and inverse transformation are calculated according to (3).

Matrix $H$ can be transformed in real numbers:

$$H = \left[ \begin{array}{cc} \text{Re}(H) & -\text{Im}(H) \\ \text{Im}(H) & \text{Re}(H) \end{array} \right]$$

$$H = \left[ \begin{array}{c} z \\ \sqrt{(n,n)} I_{2N} \end{array} \right] = \left[ \begin{array}{c} \text{Re}(z) \\ \text{Im}(z) \end{array} \right]$$

(9)

where $I_{2N}$ is the $2N \times 2N$ identity matrix, and $0_{2N}$ is the $2N \times 1$ zero vector.

In this case equation (1) can be rewritten as:

$$z = HS + n$$

(10)

Lenstra-Lenstra-Lovazh algorithm can produce matrices

$$H = H^{\text{red}} T^{-1} = QRT^{-1}$$

(11)

Thus, operation (6) in the introduced notations can be carried out as:

$$z = \left( \begin{array}{cc} \text{Re}(H)^T & -\text{Im}(H)^T \\ \text{Im}(H)^T & \text{Re}(H)^T \end{array} \right) \left( \begin{array}{c} z \\ \sqrt{(n,n)} I_{2N} \end{array} \right)$$

A transition to integer numbers can be performed as

$$\bar{z} = \frac{1}{2A} x - \frac{1}{2} T^{-1} 1_{2N}$$

(12)

where $1_{2N}$ is the $2N \times 1$ unit vector.

The reason of operation (12) is to change counting in (2) from $1,3,5,7$ ... to $0,1,2,3$ ... taking into account $\pm$ sign. Then operation (7) can be completed as

$$\bar{s} = 2AT\bar{c} + A1_{2N}$$

(13)

And the inverse transformation from integer numbers obtained as a result of transformation (12) to real numbers (as well as from $0,1,2,3$ ... back to $1,3,5,7$ ...) can be performed according to the formula:

$$\bar{s} = 2AT\bar{c} + A1_{2N}$$

(14)

Finally, a transition from $\bar{s}$ to $s$ is carried out inverse to the reason, which is mentioned in formulas (9). The results of this algorithm simulation are shown in Fig. 2, 3. It is seen from the
figures that the proposed algorithm ensures significantly better reception performance over MMSE, but is notably inferior to MLA and QRM.

SIC-LR-MMSE
The reception performance can be further improved by using successive interference cancellation (SIC) methods provided in the literature, for example, [4,5,9].

To do this, interference in the observation vector \( \mathbf{z} \) can be reduced to the triangular form by transformation

\[
y = Q^T \mathbf{z}.
\]

(15)

At the same time prior to performing (11), matrix \( \mathbf{H} \) columns should be rearranged in the order of decreasing diagonal elements module of matrix \( \mathbf{F}^{-1} \) for sorting purposes.

The initial estimation of transmitted data symbols can be obtained from (13) in notations \( T^{-1} \mathbf{z} \), that is:

\[
\mathbf{g} = T^{-1} \mathbf{z} = 2A \mathbf{c} + A \mathbf{T}^{-1} \mathbf{1}_{2N}.
\]

(16)

Then vector \( y \) is iteratively cleared from multiple access interference, namely

\[
y[2N-j+1] = y[2N-j+1] - \sum_{k=1}^{j} R[2N-j+1,2N-k+1] \mathbf{g}[2N-k+1]
\]

(17)

where \( j = \frac{1}{2}2N \)

Afterwards (17) the derived values should be normalized by the diagonal elements of matrix \( R \).

\[
y[2N-j+1] = \frac{y[2N-j+1]}{R[2N-j+1,2N-j+1]}
\]

(18)

Once transformations (17), (18) are carried out, estimation of the transmitted data symbols \( \mathbf{z} \) can be made by applying standard formulas (12) and (14):

\[
\mathbf{z} = 2A \mathbf{T} \left[ \frac{1}{2} y - \frac{1}{2} \mathbf{T}^{-1} \mathbf{1}_{2N} \right] + A \mathbf{1}_{2N}
\]

(19)

PIC-LR-MMSE
A PIC canceller is based on methods, listed in [4,5].

This canceller is applied directly to the observed signal (1) without performing transformations (3)-(6).

If the output of MMSE (7) or LR-MMSE (11)-(14) algorithm is already available, let us denote “soft” values of any of these algorithms as \( s^{(0)} \). These values are initial values of the PIC canceller (zero iteration).

If a PIC method is utilized after the MMSE algorithm, values (7), that is, the output of the matched filter, can be used as an initial value of the PIC method:

\[
s^{(0)} \equiv s = W_{MMSE}^Z
\]

(20)

The Jakes fading model [11], a multipath model “Vehicular A” according to [12,13] were used in simulations. The bandwidth is 10 MHz [14], carrier frequency is 2.4 GHz. OFDM scheme is used to cancel multipath interference [2]. Fast Fourier Transform (FFT) size is selected to 1024 symbols (carrier spacing is approximately equal to 10 kHz). The cyclic prefix value is selected more than maximum possible propagation delay value, so multipath interference is considered as negligibly small.

III. SIMULATION RESULTS
The results of the proposed SIC algorithm simulation are shown in Fig. 4.

In the figures, “PIC-MMSE 1 iteration” and “PIC-MMSE 3 iterations” means using one and three iteration of the PIC canceller with MMSE correspondingly. “PIC-LR-MMSE 3 iterations” means 3 iterations of PIC with LR-MMSE.

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As it is seen from figures, PIC- and SIC- MMSE methods outperform by performance MMSE algorithm. At the same time PIC- and SIC-LR-MMSE outperform LR-MMSE one. But SIC-MMSE and PIC-MMSE do not exceed LR-MMSE algorithm by characteristics. So, for demodulators with critical requirements to calculation complexity is possible to use simplified versions like PIC-MMSE and SIC-MMSE instead of LR-MMSE with some gain losses, because complexity of PIC-MMSE and SIC-MMSE is less than LR-MMSE.

Easy to show, that MMSE and LR-MMSE algorithms require \( \sim N^3 \) operations of multiplication from (7) – two matrix multiplications and one matrix inversion. At the same time both PIC and SIC methods require only \( \sim N^2 \) operations (for PIC case \( \sim N^2 N_t \)). So, for moderate number of iterations PIC and SIC approaches require less complexity, than LR-MMSE.

V. CONCLUSION

A variant of the PIC and SIC algorithms with classical MMSE and as well as with LR-MMSE is presented. Comparison of SIC-LR-MMSE and PIC-LR-MMSE to QRM and MLA is made. It is shown, that proposed algorithms with square computational complexity depending on the number of antennas and modulation order outperform conventional algorithms by characteristics, i.e. SIC-LR-MMSE and PIC-LR-MMSE have characteristics, close to potentially achievable, estimated by QRM and MLA ones. The gain of PIC-LR-MMSE and SIC-LR-MMSE algorithms over LR-MMSE and PIC-MMSE and SIC-MMSE over MMSE is about 2 dB for fading propagation channel conditions, used in current article.

REFERENCES


