Robust composite nonlinear feedback for nonlinear Steer-by-Wire vehicle’s Yaw control

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Abstract

Yaw control is a part of an Active Front Steering (AFS) system, which is used to improve vehicle manoeuvrability. Previously, it has been reported that the yaw rate tracking performance of a linear Steer-by-Wire (SBW) vehicle equipped with a Composite Nonlinear Feedback (CNF) controller and a Disturbance Observer (DOB) is robust with respect to side wind disturbance effects. This paper presents further investigation regarding the robustness of the combination between a CNF and a DOB in a nonlinear environment through a developed 7-DOF nonlinear SBW vehicle. Moreover, in contrast to previous studies, this paper also contributes in presenting the validation works of the proposed control system in a real-time situation using a Hardware-in-Loop (HIL) platform. Simulation and validation results show that the CNF and DOB managed to reduce the influence of the side wind disturbance in nonlinearities.

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1. INTRODUCTION

The removal of mechanical linkages between the steering wheel and the front wheel system in a Steer-by-Wire (SBW) assembly can overcome the limitations of a traditional steering system [1]. For example, an Active Front Steering (AFS) system which requires no fixed relationship between the steering and the front wheels can be implemented independently without interference by the driver [2]. AFS is commonly utilised for yaw rate tracking control to enhance the manoeuvrability of the vehicle.

A Composite Nonlinear Feedback (CNF) controller is one of the various controllers that have been applied in an AFS system. CNF has strong capability in achieving a fast yaw rate tracking performance with minimal overshoot [3-7]. Other than AFS, the CNF also performed well in active anti-roll bar and wheel synchronisation systems [8-13]. However, CNF is not robust with regard to disturbances. Thus, extensive work has been carried out to solve this issue. Hassan et al. combined a reduced-order observer into the basic CNF design to eliminate disturbances in an AFS system [14]. In 2014, Huang et al. adopted an extended state observer to estimate the unknown disturbance in a servomotor speed regulation system [15]. Hu et al. proposed an Integral Sliding Mode (ISM) based on a CNF for path tracking purposes [16]. On the other hand, Saruchi et al. introduced a combination of a CNF and a disturbance observer (DOB) to form a robust CNF controller to cater for side wind disturbance effects in yaw rate tracking control [17]. However, the robust...
CNF performance was only evaluated in a linear environment by using a linear Steer-by-Wire (SBW) vehicle system. In addition, the work has only been conducted via simulation.

The main contribution of this study is to enhance the work of Saruchi et al. [17]. This study presents further robustness analysis of the combination between a CNF and a DOB in nonlinearities using a nonlinear SBW vehicle system. This study also investigates and validates the controller’s performance in a real-time condition through a Hardware-in-Loop (HIL) platform.

2. RESEARCH METHOD

2.1. Vehicle System Model

Figure 1 shows the 7-DOF nonlinear vehicle model and a 2-DOF linear vehicle system model. Table 1 tabulates the parameters for both models. The nonlinear model is used as the vehicle plant, while the linear model is utilised in the design of the controller. Considering the existence of a side wind disturbance force $F_\omega$, the governing equations of lateral, longitudinal and yaw motions for a nonlinear vehicle system are expressed as follows:

$$m \ddot{v}_x = -mv_x \gamma + F_{yFR} + F_{yFL} + F_{yRR} + F_{yRL} + F_\omega$$  \hspace{1cm} (1)

$$m \ddot{v}_y = mv_y \gamma + F_{xFR} + F_{xFL} + F_{xRR} + F_{xRL}$$  \hspace{1cm} (2)

$$I_z \ddot{\gamma} = l_F (F_{yFR} + F_{yFL}) - l_R (F_{yRR} + F_{yRL}) + \frac{T}{2} (F_{xFL} - F_{xFR}) + \frac{T}{2} (F_{xRL} - F_{xRR}) + L \omega F_\omega$$  \hspace{1cm} (3)

where, $v_x$ and $v_y$ denote the lateral and longitudinal accelerations. Here, $F_{xi}$ and $F_{yi}$ ($i = FR, FL, RR, RL$) are the tyre forces in the $X$ and $Y$ directions respectively, which can be related to the tractive tyre force $F_{TRi}$ and lateral tyre force $F_{LTi}$. The equation can be expressed as:

$$F_{xi} = F_{TRi} \cos \delta_{fwi} - F_{LTi} \sin \delta_{fwi}$$  \hspace{1cm} (4)

$$F_{yi} = F_{TRi} \sin \delta_{fwi} + F_{LTi} \cos \delta_{fwi}$$  \hspace{1cm} (5)

where $\delta_{fw}$ is the front wheel angle. The equation of the wheel rotational motion is described as:

$$I_\omega \omega_i = -R_\omega F_{xi} + T_i$$  \hspace{1cm} (6)

where, $\omega_i$ is the wheel rotational speed, $T_i$ is the difference between the driving torque, $T_d$, and the braking torque, $T_b$.

![Figure 1. Structure of vehicle system models](image)

This model includes a quasi-static load transfer to generate the normal force generations. The normal load equation for each wheel is as follows:
\[ F_{x\text{FR,FL}} = \frac{mgF}{2L} - \frac{mvxh_{CG}}{2L} \pm \frac{mvyh_{CG}}{2T} \] (7)

\[ F_{x\text{RR,RL}} = \frac{mgR}{2L} + \frac{mvxh_{CG}}{2L} \pm \frac{mvyh_{CG}}{2T} \] (8)

Table 1. Vehicle model parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_F ) &amp; ( l_R )</td>
<td>Distance of Front &amp; Rear Wheel to COG</td>
<td>1.14 &amp; 1.64</td>
<td>m</td>
<td>( C_F ) &amp; ( C_R )</td>
<td>Front &amp; Rear Wheel Cornering Stiffness</td>
<td>54500 &amp; 42600</td>
<td>N/rad</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Body slip angle</td>
<td>-</td>
<td>rad</td>
<td>( \gamma )</td>
<td>Yaw rate</td>
<td>-</td>
<td>rad</td>
</tr>
<tr>
<td>( v_y )</td>
<td>Lateral velocity</td>
<td>-</td>
<td>ms(^{-2})</td>
<td>( I_z )</td>
<td>Yaw Inertia</td>
<td>4000</td>
<td>kgm(^2)</td>
</tr>
<tr>
<td>( v_x )</td>
<td>Longitudinal velocity</td>
<td>-</td>
<td>ms(^{-2})</td>
<td>( m )</td>
<td>Vehicle Mass</td>
<td>1529.98</td>
<td>kg</td>
</tr>
<tr>
<td>( R_\omega )</td>
<td>Wheel radius</td>
<td>0.33</td>
<td>m</td>
<td>( h_{FP} )</td>
<td>External Force Point</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>( I_\omega )</td>
<td>Wheel inertia</td>
<td>2.1</td>
<td>kgm(^2)</td>
<td>( F_w )</td>
<td>Crosswind Force</td>
<td>2000</td>
<td>Nm</td>
</tr>
<tr>
<td>( l )</td>
<td>Track width</td>
<td>1.55</td>
<td>m</td>
<td>( g )</td>
<td>Gravity</td>
<td>9.8</td>
<td>ms(^{-2})</td>
</tr>
<tr>
<td>( a_i )</td>
<td>Slip angle</td>
<td>-</td>
<td>rad</td>
<td>( h_{CG} )</td>
<td>Height of c.g.</td>
<td>0.54</td>
<td>m</td>
</tr>
</tbody>
</table>

Then, the well-known Pacejka’s Magic Formula tyre model is used to obtain the responses of the nonlinear vehicle longitudinal and lateral forces [18]. On the other hand, the governing equations for the lateral and yaw motions of the linear vehicle model which is linearised from the nonlinear vehicle can be expressed as follows:

\[ m v (\beta + \gamma) = -2C_F (\beta + \frac{v_y}{v} \gamma - \delta_{fw}) - 2C_R (\beta - \frac{v_y}{v} \gamma) + F_w \] (9)

\[ I_z \gamma = -2I_F C_F (\beta + \frac{v_y}{v} \gamma - \delta_{fw}) + 2I_R C_R (\beta - \frac{v_y}{v} \gamma) + I_\omega F_w \] (10)

### 2.2. Front Wheel System model

Figure 2 shows the front wheel system (FWS) configuration while Table 2 lists its parameters [19]. Basically, the FWS consists of a steering wheel, motor, rack, pinion and front wheels [20]. Input and output for the FWS system is the steering wheel angle \( \delta_{sw} = V \) and front wheel angle \( \delta_{fw} \).

![Mechanism](a) Mechanism ![Block diagram](b) Block diagram

Figure 2. Front wheel system configuration

With reference to Figure 1, the mathematical model for FWS can be expressed as:

Motor Displacement and Current:

\[ \dot{\theta}_m = -\frac{1}{J_m} (B_m \dot{\theta}_m + K_i i) , \quad i = \frac{1}{L} (-R_1 - K_b \dot{\theta}_m + V) \] (11)

Robust composite nonlinear feedback for nonlinear Steer-by-Wire vehicle’s Yaw... (Sarah ‘Atifah Saruchi)
Rack and pinion:

\[
\ddot{y}_r = \frac{1}{m_r} \left( -\frac{2K_l}{r_f} \dot{y}_r - \frac{K_r}{r_p} \dot{y}_r - B_r \dot{y}_r - \frac{K_r}{r_p} \theta_m \right)
\]  

(12)

Front wheel displacement:

\[
\ddot{\delta}_{fw} = \frac{1}{f_r} \left( -K_l \left( \delta_{fw} - \frac{2\gamma}{r_f} \right) - B_k \dot{\delta}_{fw} \right)
\]  

(13)

(11), (12) and (13) can be represented in the following transfer function forms,

\[
G_1(s) = \frac{(-2.274 \times 10^{-3})s^2 + (2.365 \times 10^{-11})s}{s^3 + 506.2s^2 + 11360s + 34390}
\]  

(14)

\[
G_2(s) = \frac{(-3.553 \times 10^{-15})s + (1.75 \times 10^4)}{s^2 + 12.5s + (1.805 \times 10^3)}
\]  

(15)

\[
G_3(s) = \frac{(1.421 \times 10^{-14})s + 1015}{s^2 + 51.47s + 1471}
\]  

(16)

Table 2. Front Wheel System parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_m)</td>
<td>Motor angle</td>
<td>-</td>
<td>rad</td>
<td>(m_r)</td>
<td>Rack Mass</td>
<td>2</td>
<td>kg</td>
</tr>
<tr>
<td>(V)</td>
<td>Voltage</td>
<td>-</td>
<td>V</td>
<td>(r_p)</td>
<td>Pinion gear radius</td>
<td>0.1</td>
<td>m</td>
</tr>
<tr>
<td>(i)</td>
<td>Motor current</td>
<td>-</td>
<td>A</td>
<td>(J_m)</td>
<td>Motor inertia</td>
<td>0.0012</td>
<td>kgm</td>
</tr>
<tr>
<td>(y_r)</td>
<td>Rack displacement</td>
<td>-</td>
<td>m</td>
<td>(J_f)</td>
<td>Front wheel inertia</td>
<td>1.36</td>
<td>kgm</td>
</tr>
<tr>
<td>(m_r)</td>
<td>Rack Mass</td>
<td>2</td>
<td>kg</td>
<td>(J_m)</td>
<td>Motor inertia</td>
<td>0.0012</td>
<td>kgm</td>
</tr>
<tr>
<td>(B_m)</td>
<td>Motor damping coeff.</td>
<td>0.007</td>
<td>Nms/rad</td>
<td>(L)</td>
<td>Motor electrical induct.</td>
<td>1.6663</td>
<td>Henry</td>
</tr>
<tr>
<td>(K_l)</td>
<td>Lumped torque stiff.</td>
<td>0.257</td>
<td>Nm/rad</td>
<td>(R)</td>
<td>Motor electrical resist.</td>
<td>3.1124</td>
<td>Ohm</td>
</tr>
<tr>
<td>(K_e)</td>
<td>Motor emf constant</td>
<td>0.2319</td>
<td>V/s</td>
<td>(r)</td>
<td>Offset of king pin axis</td>
<td>0.69</td>
<td>m</td>
</tr>
<tr>
<td>(K_s)</td>
<td>Steering linkage stiff.</td>
<td>2.600</td>
<td>Nms/rad</td>
<td>(B_m)</td>
<td>Motor damping coeff.</td>
<td>0.007</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>(B_k)</td>
<td>Rack damping coeff.</td>
<td>25</td>
<td>Nms/rad</td>
<td>(B_k)</td>
<td>King pin damping coeff.</td>
<td>70</td>
<td>Nms/rad</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* stiff.=stiffness, coeff.=coefficient, induct.=inductance, resist.=resistance

2.3. Controller design

Table 3 shows parameters of the control structure. Figure 3 illustrates an overview of the control structure which includes wheel synchronisation, disturbance rejection and yaw rate tracking control systems. FWS requires a wheel synchronisation controller, \(C_1(s)\) to control the motor position to ensure that the front wheel angle, \(\delta_{fw}\) is able to synchronise with the steering input angle, \(\delta_{sw}\). Here, CNF is utilised as the controller \(C_1(s)\) [11]. In the disturbance rejection control system, a DOB is implemented to eliminate the side wind disturbance effect, \(\omega\) without affecting the steering wheel. In the yaw rate tracking control system, an additional corrected angle, \(\delta_c\) obtained from the CNF controller \(C_2(s)\) is added to the steering input, \(\delta_{sw}\) to keep the actual yaw rate response, \(\gamma\) closely tracking the desired yaw rate response, \(\gamma_{ref}\) generated by the reference model, \(R_2(s)\). Detailed descriptions of the CNF and the DOB designs are explained in [17].

Table 3. Parameters of the control structure

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\omega})</td>
<td>Estimated side wind disturbance</td>
<td>(G_2(s))</td>
<td>Transfer function of nominal yaw model</td>
</tr>
<tr>
<td>(\theta_D)</td>
<td>Disturbance compensator control input</td>
<td>(G_2^{-1}(s))</td>
<td>Inverse Transfer function of nominal yaw model</td>
</tr>
</tbody>
</table>
2.4. Simulation setup

The performance of a robust CNF (a combination of a CNF and a DOB) with regard to a nonlinear vehicle system can be evaluated in simulation using Matlab/Simulink software. As shown in Figure 4, the simulation is conducted for two types of manoeuvres which are J-curve and Lane change. Then, a side wind disturbance is purposely added to the manoeuvres to investigate the robustness of the controller. A comparison of the responses from the linear and nonlinear vehicle systems is conducted to investigate the yaw rate tracking abilities. During simulation, the vehicle is assumed to be running at a constant speed of 80 km/h under normal road conditions. In order to generate the desired yaw rate response, a reference model is derived as the following (17) [21, 22]:

$$\gamma_{ref} = \frac{K_r}{1 + T_s S} \delta_{fwref}$$  (17)

where,

$$K_r = \frac{v}{L(1 + K_s v^2)}, \delta_{fwref} = \frac{n}{T_{fwS} + 1} \delta_{sw}$$

Here, $K_r$ is the stability factor, $T_s$ is the desired time constant, $\delta_{fwref}$ is the desired front wheel angle, $n$ is the steering ratio, $T_{fw}$ is the delay time and $\delta_{sw}$ is the steering wheel angle.

2.5. Hardware-in-Loop setup

Figure 5 shows the overview of the Hardware-in-Loop (HIL) platform. The platform is built using a Matlab-XPC target to validate the robustness of the CNF performance in real time. During the setup, the DC motor in the FWS is replaced by a hardware mechanism while the rest of the FWS components and vehicle model remain simulated in the main PC [23]. A crossover cable is used to communicate between the main PC and the target XPC. The target monitor shows the results of the HIL test. Signals from the main PC are sent to the DC motor. Then, the signals from the DC motor are measured by the encoder and supplied to the target XPC through the NI card.
3. RESULTS AND ANALYSIS

3.1. Robustness analysis

The effectiveness of the robust CNF performance is verified through a robustness analysis. The front and rear cornering stiffness of the vehicle model is varied to investigate the controller’s robustness. The analysis is conducted in simulation using $\pm 20\%$ of the nominal front and rear cornering stiffness parameter values ($C_F, C_R$). Figure 6 and Figure 7 depict the yaw rate tracking responses during J-curve and Lane change manoeuvres. Both figures show that the output responses are identical to the nominal responses even though the front and rear cornering stiffness values are varied within the $\pm 20\%$ range. The yaw rate responses increased by 3.2% and 4.5% respectively when the front and rear cornering stiffness changed to $+20\%$, and decreased by 6.5% and 5.2% respectively when the cornering stiffness changed to $-20\%$. However, overall, the control system is stable in both cases. Based on these results, the controller is proven to be robust under varied cornering stiffness parameters.
3.2. Simulation and experiment results

The performance analysis of the robust CNF controller in the yaw rate tracking control is carried out both in simulation and by experiment. In simulation, the control system is tested using linear and nonlinear vehicle models for comparison purposes. Then, the control system performance in the nonlinear environment is validated using a real-time HIL test. The output responses of the yaw rate tracking performance for both the simulation and experimental platforms are compared and illustrated in Figure 8(a) and 9(a). The analysis is continued by calculating the tracking error. The tracking error responses are shown in Figure 8(b) and 9(b), while the numerical results are tabulated in Table 4.

![Figure 8. Tracking responses and errors during J-curve manoeuvre](image)

![Figure 9. Tracking responses and errors during Lane change manoeuvre](image)

<table>
<thead>
<tr>
<th>Platforms</th>
<th>J-curve</th>
<th>Lane change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation (Linear)</td>
<td>$3.50 \times 10^{-4}$</td>
<td>$2.80 \times 10^{-4}$</td>
</tr>
<tr>
<td>Simulation (Nonlinear)</td>
<td>$4.22 \times 10^{-4}$</td>
<td>$3.94 \times 10^{-4}$</td>
</tr>
<tr>
<td>Experiment (Nonlinear)</td>
<td>$3.10 \times 10^{-3}$</td>
<td>$1.60 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Based on the tracking results for the simulation platform, it can be seen that the linear and nonlinear systems produced quite similar yaw rate output responses. On the other hand, the experimental results show that the produced yaw rate responses produced are able to track the desired signals even though there is a slightly higher overshoot and delay due to the model uncertainties such as backlash and noise. It is believed that the gap between the gears in the mechanical parts also leads to such responses.

According to the simulation results, the nonlinear system with a robust CNF controller managed to eliminate 91.13% and 97.02% of the influence caused by the side wind disturbance in the J-curve and Lane change manoeuvres, respectively. Meanwhile, based on experimental results, 78.91% and 87.88% of the disturbances effect are eliminated in both manoeuvres.
4. CONCLUSION
This study presents extensive work based on a previous study by Saruchi et al. [15] by further analysis of the robustness of the combination of a CNF and a DOB in nonlinearities through the development of a nonlinear vehicle model. The analysis is then verified in real-time using a HIL platform. Simulation and experimental results prove that the CNF and DOB combination manages to closely track the desired yaw rate response with the least effect from side wind disturbance in the nonlinear condition. After being tested with J-curve and Lane change manoeuvres, 97.13% and 97.02% of the disturbances effect are reduced during simulation, while, 78.91% and 87.88% of the disturbances effect are reduced during experiment.

As for future work, it is encouraged to investigate the performance of the controller under more extreme conditions such as a double lane change manoeuvre. In terms of improvement in the accuracy of the experimental results, it is recommended to set up a HIL platform with a real rack, pinion and tyres. Additionally, the issue of model uncertainties such as backlash and noise should be further investigated and solved.

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