Robust Adaptive Sliding Mode Control Design with Genetic Algorithm for Brushless DC Motor

Abstract—This study aims to design a control scheme that is capable to improve performance and efficiency of brushless DC motor (BLDC) in operating condition. The control scheme is composed of sliding mode controller (SMC) with proportional-integral-derivative (PID) sliding surface. The PID sliding surface is used to improve the system transient response. Then, the SMC-PID is optimized by genetic algorithm optimization for further improvement on the stability and robustness against nonlinearities and disturbances. Chattering problem that appear in the SMC is minimized by employing an adaptive switching gain for the SMC that is integrated with Luenberger Observer. Lyapunov function candidate is applied to guarantee the stability of the system. Simulation on the proposed work is done in Matlab Simulink. Results of the simulation works indicate that the proposed control scheme can improve the transient response, the stability and robustness of the BLDC motor compared to the conventional SMC in the existence of nonlinearities and disturbances.

Keywords—BLDC motor, sliding mode control, PID sliding surface, genetic algorithm, adaptive switching gain

I. INTRODUCTION

Brushless direct current motor (BLDCM) is one of synchronous permanent magnet DC motors. BLDCM is very different from conventional DC motor, since the BLDCM is designed without brush. Without commutation component the motor produces several advantages, such as high efficiency, reducing vulnerability in mechanical components, large torque, longer lifetime, and low noise [1]. Due to their significant role in industrial applications, improvements on the speed control are much needed. Hence, most of researches that discuss the BLDCM speed control are focus on linear and nonlinear modeling and testing with varying loads [2].

Some of the studies that have been done were speed control with fuzzy logic [3], fuzzy PID [4], and Fuzzy Genetics Algorithm [5], where the controller were tested on BLDC linear motor models. The linear model is a model that is only capable presenting the motor when in its maximum performance. But, sometimes with continuous usage it faces some problems, such as friction and the change of parameters that cause the motor no longer linear. These problems were driving several researchers to develop a speed control method based on nonlinear models such as sliding mode control (SMC) [6].

SMC is a control system that has the ability to maintain a system stability in various models with various interference and system parameters. So it is often used in nonlinear models. SMC has a working area on the steady state phase, so that when disturbance occurs and also parameter changes, SMC is able to keep the system performance stable. Recently SMC-PID has been developed, where the SMC sliding surface can be arranged with proportional, integral and derivative parameters. This is done so that the transient response of the system is better than the previous SMC. SMC-PID has proven its performance on conventional DC motors by relying solely on determining PID parameters by trial and error. Unlike conventional DC motors that tend to be easy in parameter determination, BLDCM has a more complicated structure and high nonlinearity level that requires the optimization of parameters for SMC-PID to achieve its best performance [7].

The process of optimizing the PID parameters is done to find the most suitable value to be applied to the system. The genetic algorithm (GA) is used as the optimization method in BLDCM in this research. GA was chosen because it is a modern optimization method that has a high searching capacity, and a heuristic character [8]. With the advantages of the GA method, the determination of the SMC-PID parameter value on the nonlinear BLDCM can be resolved. However, there is still a problem that arises between the advantages of SMC-PID that is the high rate of the chattering when a various disturbance with high magnitude hit the system [9].

The high chattering on the control system is due to the high switching gain of the SMC-PID that should be adjusted against the high magnitude of the disturbance present in the system. Therefore it is necessary to have a Luenberger Observer as a tool to estimate the disturbance in the BLDCM. The estimated value of the disturbance observer is then converted as a determinant of the switching gain of the SMC-PID. Thus, the value of the switching gain that estimates the magnitude of the chattering will adapt in accordance with the value of the disturbance changes received by the system. It’s an adaptive robust control scheme that relates to the magnitude of the existed disturbance in the system.

II. MODELING SYSTEM

A. State Space Modeling

Modeling is done by state space method where this method is used as a modern method. State space method is
also much done because it is easy in the application of systems that have input and output more than one and easier in terms of computing.

Suppose that the three-phase BLDC motor is controlled by the full bridge driving in the two phase conduction mode [11] as explained below.

\[ i_A + i_B + i_C = 0 \]  
(1)

\[ u_{AB} = r_a (i_A - i_B) + L_a \frac{d}{dt}(i_A - i_B) + e_{AB} \]  
(2)

\[ u_{BC} = r_a (i_A + 2i_B) + L_a \frac{d}{dt}(i_A + 2i_B) + e_{BC} \]  
(3)

Then, subtract equation (2) from equation (3)

\[ u_{AB} - u_{BC} = -3r_a - 3L_a \frac{d}{dt}i_B + e_{AB} - e_{BC} \]  
(4)

\[ i_B' = -\frac{r_a}{L_a} i_B - \frac{1}{3L_a}(u_{AB} - e_{AB}) + \frac{1}{3L_a}(u_{BC} - e_{BC}) \]  
(5)

The calculations for the second phase are performed by the same method as in equations (2) and (3).

\[ u_{AB} = r_a (2i_A + i_B) + L_a \frac{d}{dt}(2i_A + i_B) + e_{AB} \]  
(6)

\[ u_{CA} = r_a (i_C - i_A) + L_a \frac{d}{dt}(i_C - i_A) + e_{CA} \]  
(7)

Subtract equation (6) from equation (7)

\[ (u_{AB} - e_{AB}) - (u_{CA} - e_{CA}) = 3r_a + 3L_a \frac{d}{dt}i_A \]  
(8)

\[ i_A' = -\frac{r_a}{L_a} i_A + \frac{1}{3L_a}(u_{AB} - e_{AB}) - \frac{1}{3L_a}(u_{CA} - e_{CA}) \]  
(9)

\[ u_{AB} = u_{BC} \]  
(10)

\[ e_{AB} = e_{BC} \]  
(11)

\[ u_{CA} = -(u_{AB} + u_{BC}) = -2u_{AB} \]  
(12)

\[ e_{CA} = -(e_{AB} + e_{BC}) = -2e_{AB} \]  
(13)

The calculation proceeds by substituting equations (10), (11), (12) and (13) in equation (9).

\[ i_A' = -\frac{r_a}{L_a} i_A + \frac{1}{3L_a}(u_{BC} - e_{BC}) + \frac{2}{3L_a}(u_{AB} - e_{AB}) \]  
(14)

Other characteristic equation that had by BLDCM is the relation of torque and speed.

\[ K_T i - T_L = J \frac{d\omega}{dt} + B_v \omega \]  
(15)

Or it could be written in another form

\[ \omega = -\frac{B_v}{J} \omega + \frac{1}{J}(T_e - T_L) \]  
(16)

where,

\[ T_e = K_T i \]  
(17)

\[ U_d : \text{DC bus voltage.} \]

\[ e_A : \text{Phase back EMF.} \]

\[ r_a : \text{Line resistance of winding, } r_a = 2R. \]

\[ L_a : \text{Equivalent line inductance of winding, } L_a=2(L-M). \]

\[ J : \text{Rotor moment inertia.} \]

\[ T_L : \text{Load torque} \]

\[ \omega : \text{Rotor speed.} \]

\[ B_v : \text{Viscous friction coefficient.} \]

\[ K_e : \text{Coefficient of line back EMF} \]

\[ K_T : \text{Coefficient of line torque constant} \]

\[ M : \text{Mutual Linkage, assume } M=0. \]

From the equations that has been elaborated above, the dynamic equations of the BLDC motor is represented in the following state space form.

\[ \begin{bmatrix} \dot{u}_{ib} \\ \dot{u}_{ic} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & 0 & 0 \\ 0 & -\frac{r_a}{L_a} & 0 \\ 0 & 0 & -\frac{B_v}{J} \end{bmatrix} \begin{bmatrix} u_{ib} \\ u_{ic} \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{2}{3L_a} & 0 & 0 \\ 0 & \frac{2}{3L_a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{AB} - e_{AB} \\ u_{BC} - e_{BC} \end{bmatrix} \]  
(18)

where \( \theta \) is the rotor position.

From the BLDC equation will be added friction column such as non-linear factor where the factor is a friction that can inhibit the performance of the motor that causes the motor no longer work as a linear system. If column friction is added to equation (18), then the equation becomes as follows. The friction is represented as nonlinearity that exist in the system.

\[ \begin{bmatrix} \dot{u}_{ib} \\ \dot{u}_{ic} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & 0 & 0 \\ 0 & -\frac{r_a}{L_a} & 0 \\ 0 & 0 & -\frac{B_v}{J} \end{bmatrix} \begin{bmatrix} u_{ib} \\ u_{ic} \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{2}{3L_a} & 0 & 0 \\ 0 & \frac{2}{3L_a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{AB} - e_{AB} \\ u_{BC} - e_{BC} \end{bmatrix} \]  
\[ + \begin{bmatrix} 0 \\ 0 \\ \eta(\theta) \end{bmatrix} \]  
(19)

Following the complete state equations in (19) for the BLDC motor, the parameters of the motor are given as appeared in Table I. The parameters are taken from the BLDC motor parameters that was analyzed in the previous research [5].

**TABLE I. UNITS FOR BLDC Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Unit Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_a )</td>
<td>Resistance</td>
<td>0.25 Ohm</td>
</tr>
<tr>
<td>( L_a )</td>
<td>Inductance</td>
<td>0.32 mH</td>
</tr>
<tr>
<td>( J )</td>
<td>Rotor Inertia</td>
<td>0.0042 Kg-m²</td>
</tr>
<tr>
<td>( M )</td>
<td>Mutual Linkage</td>
<td>0 mH</td>
</tr>
<tr>
<td>( P )</td>
<td>Pole</td>
<td>8 poles</td>
</tr>
<tr>
<td>( B_v )</td>
<td>Friction coefficient</td>
<td>0.0096 Nm/(rad/sec)</td>
</tr>
<tr>
<td>( K_e )</td>
<td>Back EMF constant</td>
<td>0.65 V/rad/sec</td>
</tr>
<tr>
<td>( K_T )</td>
<td>Torque constant</td>
<td>0.0994 Nm/A</td>
</tr>
<tr>
<td>( F_c )</td>
<td>Friction column</td>
<td>0.02</td>
</tr>
</tbody>
</table>
B. Sliding Mode Controller

This study focuses on how to design the SMC control system for the BLDC motor that can be shown in the block diagram in Fig. 1. SMC will be the main control system that keeps motor performance from nonlinearity and disturbance. Then the PID sliding surface used which has the advantage of controlling the transient response of the system.

SMC with PID sliding surface will be applied to BLDC 2nd order system, thus yielding the following equation [10].

\[ s(t) = K_p e(t) + K_i \int e(t) + K_d \frac{de}{dt} \quad (20) \]

The error occurring due to the difference between the actual value and the desired trajectory is illustrated by (21).

\[ e(t) = r(t) - y(t) \quad (21) \]

Where \( e(t) \) is an error obtained from the difference of reference value \( r(t) \) and the system output value \( y(t) \). If the equation uses 2\textsuperscript{nd} order on the model then it can be written on the equation (22).

\[ \ddot{e}(t) = \dot{r}(t) - \dot{y}(t) \quad (22) \]

From equation (2), (16) and (17), \( \ddot{\omega} \) can be described as follows:

\[ \ddot{\omega} = -A\dot{\omega} - B\omega + Cu(t) - F(t) \quad (24) \]

\[ u(t) = v(t) \quad (25) \]

\[ A = -\frac{B_s}{J} \quad (26) \]

\[ B = -\frac{Kt^2}{JLa} \quad (27) \]

\[ C = -\frac{Kt}{JLa} \quad (28) \]

\[ F(t) = Te - T_l \quad (29) \]

In general, SMC is described as equation that sums between switching control and equivalent control. Switching control is an adjustment equation when the value of \( s(t) \) ≠ 0 denoted by symbol \( U_{sw} \) and equivalent control is the adjustment equation when \( s(t) = 0 \).

\[ U_{SMC} = U_{eq} + U_{sw} \quad (30) \]

Because of the value of \( s(t) \) is equal with 0 it can be assumed that \( s(t) = \dot{s}(t) = 0 \) so the derivative of the \( s(t) \) can be written as follows:

\[ \dot{s}(t) = K_p \dot{e}(t) + K_i e(t) + K_d \ddot{e}(t) \quad (31) \]

Then it is assumed that the value of the load is 0, and equation (24) is input to change the value of \( \ddot{e}(t) \) it can be written as follows:

\[ \ddot{s}(t) = K_p \dot{e}(t) + K_i e(t) + K_d (\dot{r}(t) + A\dot{\omega} + B\omega - Cu(t)) \quad (32) \]

When \( \dot{s}(t) = 0 \), the equivalent control of SMC can be described as

\[ \dot{U}_{eq} = (KdC)^{-1}(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\ddot{\omega} + B\ddot{\omega})) \quad (33) \]

From analysis of Lyapunov stability theory that has been verified, this controller will follow Lyapunov function in equation (34).

\[ V(t) = \frac{1}{2} \dot{s}^2 \quad (34) \]

Where \( V(t) > 0 \) and \( V(0) = 0 \) for \( s(t) \neq 0 \). The achievement condition will be illustrated in the equation (35).

\[ \dot{V}(t) = s(t) \dot{s}(t) < 0 \quad ; \quad s(t) \neq 0 \quad (35) \]

This equation has purpose to ensure the value moving from reaching condition to the sliding phase under stable condition, \( \dot{V}(t) \) must be negative to guarantee the control stability.

\[ 0 > s(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\ddot{\omega} + B\ddot{\omega} - Cu(t))) \quad (36) \]

\[ 0 > s(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\ddot{\omega} + B\ddot{\omega}) - Kd Cu(t)) \quad (37) \]

To make sure equation (36) is negative, then the specified of \( u_{sw} \) is

\[ U_{sw} = sign(s)(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\ddot{\omega} + B\ddot{\omega}) - Kd C(KdC)^{-1}(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\ddot{\omega} + B\ddot{\omega}))) \quad (37) \]
By applying the sign function on the sliding surface so that the equation will be obtained for its switching control:

\[ U_{sw} = K_s \text{sat}\left(\frac{S}{\varphi}\right) \]  (38)

\[ \text{sat}\left(\frac{S}{\varphi}\right) = \left(\frac{S}{\varphi}\right) \quad \text{if} \quad \left|\frac{S}{\varphi}\right| \leq 1 \]  (39)

\[ \text{sat}\left(\frac{S}{\varphi}\right) = \text{sign}\left(\frac{S}{\varphi}\right) \quad \text{if} \quad \left|\frac{S}{\varphi}\right| > 1 \]  (40)

The equation (38), (39), and (40) in practice has a work like prediction control ideal relay. So in this case the sign function can be replaced with the hyperbolic tangent function to improve the performance of hitting control. Where the value of Ks is the gain of the switching control and \( \varphi \) is the thickness of the boundary layer. So the switching equation can be written:

\[ U_{sw} = K_s \tanh\left(\frac{S}{\varphi}\right) \]  (41)

To ensure the stability of the proposed control, the Lyapunov function candidate is applied. Various effects of discontinuous function have been reduced by substituting the hyperbolic tangent function with the boundary layer of \( \varphi \). Results of \( U_{smc} \) in equations (33) and (41) are substituted into equation (30), and (30) is rewritten as follows.

\[ U_{smc} = (KdC)^{-1}(Kp \dot{e}(t) + Ki e(t) + Kd (\ddot{r}(t) + A\omega + B\omega)) + Ks \tanh\left(\frac{S}{\varphi}\right) \]  (42)

C. Genetic Algorithm

Genetic algorithm (GA) is an optimization algorithm that utilizes genetic and natural selection mechanisms. GA operates with a set of candidate solutions or chromosomes known as the population. Each chromosome consists of a number of numbers that present the solution and can be a binary number.

Population initialization is done to generate the initial solution of a genetic algorithm problem. This initialization is done randomly as much as the desired number of chromosomes / population. Next is calculated the value of fitness and so on is done by using Roulette wheel method, tournament or ranking. Values that have a high fitness will survive in the next generation as Parent. To produce a new generation performed several operations such as selection, crossover and mutation. The procedure will be repeated until found the most optimal solution or after the desired number of generations has been met.

The value of PID which is the coefficient of Kp, Ki and Kd will be optimized by GA method to ensure maximum control. For GA initialization, we have to define some initial values. Because the performance of a good system control design will depend on how well we determine the initial parameters. The initial parameters of GA are listed in Table II.

The fitness calculations of each chromosome as the selection of objective functions are very important thing. In this study we use SSTE to calculate performance index on controller with equation as follows:

\[ \text{SSTE} = \frac{1}{N} \sum_{t=0}^{N} (e)^2/N \]  (43)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation</td>
<td>50</td>
</tr>
<tr>
<td>Population Size</td>
<td>50</td>
</tr>
<tr>
<td>Crossover Method</td>
<td>Crossover Scattered</td>
</tr>
<tr>
<td>Maximum Number of Generation</td>
<td>0.8</td>
</tr>
<tr>
<td>Selection Method</td>
<td>Tournament</td>
</tr>
<tr>
<td>Crossover Probability</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation Type</td>
<td>Uniform Mutation</td>
</tr>
<tr>
<td>Mutation Probability</td>
<td>0.1</td>
</tr>
</tbody>
</table>

D. Luenberger Observer

A linear system that uses a control system must have feedback for comparison with the reference value that will be determine the input for the control system. Feedback from the system can be set according to what reference wanted to be given. But to ensure that the feedback output corresponds to the state of the system, it needs a disturbance observer like Luenberger Observer [7].

The problem that arises in SMC control is the existence of a chattering where the value must be adjusted to the biggest disturbance to be received by the motor. This causes the system to perform a large catering even in the minor disturbances. Therefore used Observer that serves to detect disturbance on the motor and then converted to the value of switching gain (Ks) on SMC so that the value of chattering will adapt in accordance with the value of disturbance that is received by the plant. Luenberger Observer basic equation is described as follow [12]:

\[ \dot{x} = A\dot{x} + Bu + L(y - C\dot{x}) \]  (44)

with estimation error observer is defined as:

\[ \dot{\hat{x}} = \hat{x} - x \]  (45)

\[ \dot{\hat{e}} = (A - LC)e - Ed \]  (46)

\[ A - LC = \begin{bmatrix} \frac{-r_a}{La} & 0 & 0 \\ 0 & \frac{-r_a}{La} & 0 \\ 0 & 0 & -Bv \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \]  (47)

\[ \lambda I - (A - LC) = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{-r_a}{La} & 0 & -L_1 \\ 0 & \frac{-r_a}{La} & -L_2 \\ 0 & 0 & -Bv - L_3 \end{bmatrix} \]  (48)

The polynomial characteristic equation of the Luenberger Observer is the determinant of the matrix in (48).
where,
\[ \rho = -ra/La \]
\[ \sigma = -Bv/f \]

According to the Vieta equation law, the value of the root of the cubic equation is defined as:
\[ -\left( \lambda_1 \lambda_2 \lambda_3 \right) = -\rho^2 \sigma + \rho^2 L3 \]
\[ L3 = \frac{-\left( \lambda_1 \lambda_2 \lambda_3 \right) + -\rho^2 \sigma}{\rho^2} \]

Hence, we get the value of L in state Observer
\[ L = \begin{bmatrix} 0 & 0 & -\lambda_1 \lambda_2 \lambda_3 + \rho^2 \sigma \\ 0 & 0 & \rho^2 \end{bmatrix} \]

where the eigen value of \((A-LC)\) is
\[ eig(A - LC) = [\lambda_1, \lambda_2, \lambda_3] \]

The difference between the Observer's output value and the actual speed of the motor was used to determine the value of the switching gain in the SMC as shown in the equation (56).
\[ U_{sw} = (K_{observer} + Ks) \tanh \left( \frac{S}{\rho} \right) \]

### III. RESULTS AND DISCUSSION

#### A. SMC-PID-GA in BLDC Motor

Evaluation on the nonlinear BLDC motor was done by equipped with 0.5 N load at \( t = 2 \) sec and noise at \( t = 3 \). The noise and the load values influenced the motor characteristics and it was not more than 40% of the input reference.

Fig. 2 shows that the performance of the SMC-PID was significantly leading than the PID in both transient and steady state area. It was proven when disturbance occurred at \( t = 2 \) second, the SMC-PID keep the system to remain at its reference input and avoid from load perturbation. Similarly, when noise occurs, SMC-PID is able to overcome the interference, such that the system remains in stable condition. Differ from the PID controller that has a big change when the system get the load and noise occurs to the system. This is happened because there was interference on the system. The SMC-PID chattered when the system trajectory was forced to remain on the sliding surface. The SMC-PID control signal as shown in Fig. 3 will remain in chattering condition during the interference, when the interference value is more than the value of switching gain.

Optimizing the value of PID parameters using GA is highly visible from Fig. 2, where the transient response of the system works more optimally although its action control just has a little bit different from the SMC-PID without optimization. In Table III, it is found that the overshoot and error values of the system that generate by GA optimization are smaller than other methods. Compared with previous studies using only conventional SMC on BLDC motors [6], SMC-PID-GA performance is much better especially on the transient response area. Conventional SMC only considers the steady state area aspect, while SMC-PID-GA is developed in addition to steady state performance, the transient response is also noticed, so that all the things that describe the performance of the system on various conditions can be resolved.

#### B. Adaptive SMC-PID-GA

Tests on adaptive SMC will be aimed at the changing of chattering value that will occur in SMC. Luenberger observer will set the value of SMC switching gain so that the
system will perform the chattering according to the number of disturbance that received.

With the observer on the system, it appears that the system will perform the chattering according to the number of disturbance that received.

IV. CONCLUSION

A robust and adaptive control scheme has been designed and simulated successfully for BLDC motor system. The SMC with PID sliding surface be able to maintain the stability of the system according with the reference point against the disturbance and noise interference. With the addition of genetic algorithm also capable to optimize the SMC-PID performance in area of transient response. Moreover, the inclusion of the Luenberger disturbance observer in the control scheme offer a solution on the problem how to minimize chattering effect on the system, when the disturbance and noise interference changing with uncertain magnitude.

REFERENCES